FERMI GAS MODEL

It is a statistical model of the nucleus. This model pictures the nucleus as a degenerate gas of protons and neutrons much like the free electron gas in metals. The gas is considered degenerate because all the particles are crowed into the lowest possible states in a manner consistent with the requirement of Pauli exclusion principle. In this case the nature of the microscopic particles is fully reflected in its effect on the ensemble as a whole.

Nucleons are fermions having spin ½. Thus the behaviour of the neutron or the proton gas will be determined by Fermi-Dirac statistics. In such a gas at 0 K, all the energy levels upto a maximum, known as Fermi energy E_F are occupied by the particles, each level being occupied by two particles with opposite spins.

Neglecting for the moment, the electrostatic charge of the protons and supposing that the nucleus has $N = Z = A/2$. The nucleons move freely within a spherical potential well of the proper diameter with depth adjusted so that the Fermi energy raises the highest lying nucleons upto the observed binding energies. The potential well is filled separately with nucleons of each type, allowing just two particles of a given type with opposite spin to each cell in phase space of volume h^3. According to Fermi-Dirac statistics, the number of neutron states per unit momentum interval is

$$\frac{dN}{dp} = \frac{2\times 4\pi\, p^2 V}{(2\pi\hbar)^2} = \frac{Vp^2}{\pi^2\,\hbar^3}$$

where V is the volume of the nucleus. If $p_f\left[= (2M\;E_F)^{\frac{1}{2}}\right]$ is the limiting momentum below which all the states all filled. Obviously, the number of neutrons occupying momentum states upto this maximum momentum is obtained as

$$N = \int_0^{p_f} \frac{dN}{dp}\,dp = \frac{V}{3\pi^2\,\hbar^2}\,p_f^3$$

we have

$$p_f = (3\pi^2)^{\frac{1}{3}}\,\hbar\left(\frac{N}{V}\right)^{\frac{1}{3}}$$

Therefore

$$E_f = \frac{p_f^2}{2M} = (3\pi)^{\frac{2}{3}}\,\frac{\hbar^2}{2M}\left(\frac{N}{V}\right)^{\frac{2}{3}}$$

$$= \frac{\hbar^2}{2M}\left(\frac{3\pi^2 N}{V}\right)^{\frac{2}{3}} \qquad \text{------(3)}$$

where $V = \frac{4}{3}\pi\, r_0^3\, A$ is the nuclear volume which contains N particles (fermions) and M is the nucleonic mass.

We have two different types of Fermi gas in the nucleus (i) the proton gas and (ii) the neutron gas. The respective numbers of protons and neutrons are Z and A - Z. Now, assuming that the number of nucleonic states to be equal to the nucleon number in each case, one obtains the density of states for the two gases as

$$n_p = \frac{Z}{V} = \frac{Z}{\frac{4}{3}\pi\, r_0^3\, A} = \frac{3Z}{4\pi\, r_0\, A}$$

$$n_n = \frac{A-Z}{V} = \frac{3(A-Z)}{4\pi\, r_0\, A}$$

We have $r_0 = 1.2$ fm. One obtains after assuming $N = A - Z = \frac{A}{2}$

$$n_p = n_n = \frac{\frac{3}{2}}{4\pi(1.2)^3} = 0.069\ \frac{\text{nucleons}}{\text{m}^3}$$

$\therefore$ Nucleon density $\qquad N = n_p + n_n = 0.138\ \frac{\text{nucleons}}{\text{m}^3}$

Remembering that each state can be occupied by nucleons of opposite spins and substituting the above in (3), one obtains

$$E_f = \frac{\hbar^2}{2M}\left(\frac{3}{2}\pi^2 n_p\right)^{\frac{2}{3}} = 21 \text{ MeV}$$

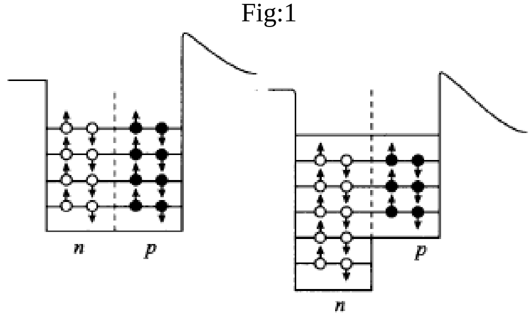
However, the number of protons (Z) and neutrons *(A* - Z) in an actual nucleus are not equal and hence *N* being somewhat greater than Z. Obviously, the Fermi energies of the two types of nucleons are different. Now $N > Z$, $(E_f)n > (E_f)p$ and hence the potential wells for the protons and neutrons have different depths, i.e. the former being less deep than the latter. The depth of the potential well is obtained as

$$V_0 = E_f + \frac{E_B}{A} = E_f + f_B$$

Here $f_B = E_B/A$ is called the mean binding energy per nucleon (binding fraction) and is of the order of 8 MeV/nucleon for both protons and neutrons. Figure 5.1(b) exhibit Fermi gas model of nucleonic potential wells.

Figure exhibits the difference in the depths of wells for neutron and proton.

Fig:1



From Figure 5.1(b), we note that the Fermi energies for both protons and neutrons are represented by the same horizontal line, corresponding to about 8 MeV below the top of the potential well (Coulomb effect is neglected). One can visualize this that if these are at different depths below the top of the well, then the nucleons of one type from the higher Fermi level (say, neutrons) would make spontaneous transitions to the lower Fermi level for the other type (protons) by beta transformations. Obviously, the levels would ultimately equalize.

Thus one finds the depth of the potential well approximately,

$$V_o \sim 21 + 8 = 29 \text{ MeV}$$

We must note that the neutron depth is slightly greater than the proton depth.

Let us now consider a hypothetical infinite medium of nuclear matter of uniform density in which the numbers of neutrons and protons are equal, i.e. $N = Z$ and the Coulomb interaction of the proton is considered negligible. In this situation, one obtains from the semi-empirical binding energy Bethe-Weizsackar relation

$$\left(\frac{E_B}{A}\right) = \left(\frac{E_\upsilon}{A}\right) = 15.9 \text{ MeV/nucleon}$$

Where E_B is binding energy. Adding this to the depth of the potential well below the Fermi level, one obtains the depth of the potential well as

$V_o = 21 + 15.9 = 36.9$ MeV/nucleon

One expect that any successful theory of nuclear matter should be able to correlate the above value of V_o to the nature of internucleon nuclear force.

We have so far assumed nuclear temperature to be $T = 0$ K corresponding to the ground state. When some excitation energy is supplied to the nucleus, then the thermal energy of the nucleus corresponds to $T > 0$ K. In this case, one can easily show that the total excitation energy is

$E_t = E_p + E_n = 11(kT)^2$ MeV

Since $kT \sim 1$ and hence $E_t \sim 11$ MeV.

One can also obtain the energy density of the nuclear levels for a given excitation energy using the entropy relation: $S = k \ln W$ and thermodynamic expression for entropy.

One cannot predict detailed properties of low lying states of nuclei observed in the radioactive decay processes from this model. This model is particularly useful in describing phenomena which are sensitive to the high momentum part of the nucleon spectrum. The model suggests that nucleon collisions often do not transfer small amount of momentum to the nucleus, because the nucleon momentum states near the origin are filled. However, this limitation does not affect collisions in which large momentum transfer takes place. Obviously, this statistical model helps to explain the properties of the nucleus in excited states. One can also treat the unbound states of heavy and medium nuclei with the help of this model.